Fundamental Mathematics and Physics Concepts for Drivetrain Analysis

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1 Newton's Laws of Motion

First Law (Inertia)

The Newton's first law of motion describes the natural tendency of objects to maintain their state of motion. Without external forces, moving objects continue moving at constant velocity, and stationary objects remain at rest.

Key Equation

$$\sum \mathbf{F} = 0 \iff \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 0$$

Second Law (Acceleration)

The Newton's second law of motion quantifies how forces cause acceleration. It establishes the direct relationship between applied force, object mass, and resulting acceleration.

Key Equation

$$\mathbf{F} = m\mathbf{a} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}(m\mathbf{v})}{\mathrm{d}t}$$

For variable mass systems:

$$\mathbf{F} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \mathbf{v}\frac{\mathrm{d}m}{\mathrm{d}t}.$$

Third Law (Action–Reaction)

The Newton's third law of motion states that forces always occur in pairs. When object A exerts a force on object B, object B simultaneously exerts an equal and opposite force on object A.

Key Equation

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

2 Kinematic Quantities

Displacement

Displacement measures the change in position of an object from one point to another. Unlike distance, displacement is a vector quantity that considers both magnitude and direction. In rotational systems, angular displacement measures the change in rotational position.

$$\mathbf{s} = \mathbf{r}(t_2) - \mathbf{r}(t_1), \qquad \theta = \theta(t_2) - \theta(t_1)$$

Velocity

Velocity describes how quickly displacement changes with time. It indicates both the speed and direction of motion. Angular velocity describes how quickly an object rotates. The relationship $v = r\omega$ connects linear and angular velocities for points on rotating objects.

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t}, \qquad \omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}, \qquad v = r\omega$$

Acceleration

Acceleration measures how quickly velocity changes with time. It occurs when an object speeds up, slows down, or changes direction. In circular motion, acceleration has two components: tangential acceleration (changing speed) and centripetal acceleration (changing direction).

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{s}}{\mathrm{d}t^2}, \qquad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

For circular motion:

$$a_t = r\alpha, \quad a_c = \frac{v^2}{r} = r\omega^2, \quad \mathbf{a} = a_t\hat{e}_t + a_c\hat{e}_r$$

3 Dynamic Quantities

Force

Force is a vector quantity that causes acceleration of objects with mass. Forces can be contact forces (friction, normal forces) or field forces (gravity, electromagnetic).

SI unit: $\mathrm{N} = \mathrm{kg\,m\,s^{-2}}$.

$$f = \mu N, \quad f \leq \mu_s N, \quad F_g = mg, \quad F_s = kx$$

Mass

Mass is a scalar measure of an object's inertia—its resistance to acceleration when subjected to force. Mass remains constant regardless of location, unlike weight which varies with gravitational field strength.

Moment of Inertia

Moment of inertia is the rotational analog of mass. It measures an object's resistance to angular acceleration about a specific axis. The distribution of mass relative to the rotation axis determines the moment of inertia value. Objects with mass concentrated far from the rotation axis have larger moments of inertia.

$$I = \sum mr^2 \quad \text{or} \quad I = \int r^2 \, dm$$

- Solid cylinder: $I = \frac{1}{2}mR^2$
- Hollow cylinder: $I = \frac{1}{2}m(R_1^2 + R_2^2)$
- Solid sphere: $I = \frac{2}{5}mR^2$

Torque

Torque is the rotational analog of force. It causes angular acceleration of objects with rotational inertia. Torque depends on both the applied force magnitude and the perpendicular distance from the rotation axis to the force application point (moment arm).

$$\tau = \mathbf{r} \times \mathbf{F}, \quad \tau = rF\sin\phi, \quad \tau = I\alpha$$

4 Energy Concepts

Work

Work represents energy transfer that occurs when a force acts through a displacement. Work is done only when the force component is parallel to the displacement direction. In rotational systems, work is done when torque acts through an angular displacement.

$$W = \int \mathbf{F} \cdot d\mathbf{s}, \quad W = Fs\cos\phi, \quad W = \int \tau \, d\theta$$

Kinetic and Potential Energy

Kinetic energy is the energy of motion—both translational and rotational motion contribute to total kinetic energy. Potential energy is stored energy due to position in a force field (gravitational) or due to deformation (elastic springs). Energy can be converted between these forms but total energy is conserved in isolated systems.

$$KE_t = \frac{1}{2}mv^2, \quad KE_r = \frac{1}{2}I\omega^2, \quad PE_g = mgh, \quad PE_s = \frac{1}{2}kx^2$$

Work–Energy Theorem

The work-energy theorem provides a powerful analysis tool by relating the net work done on an object to its change in kinetic energy.

Key Equation

$$W_{\text{net}} = \Delta KE = KE_2 - KE_1$$

Power

Power measures the rate of energy transfer or work done. In mechanical systems, power equals force times velocity for linear motion, or torque times angular velocity for rotational motion.

$$P = \frac{\mathrm{d}W}{\mathrm{d}dt}, \quad P = \mathbf{F} \cdot \mathbf{v}, \quad P = \tau\omega$$

5 Mathematical Foundations: Calculus and Physical Meaning

Calculus provides the mathematical tools to describe how quantities change and accumulate over time.

Differentiation: Measuring Rates of Change

Differentiation measures how quickly one quantity changes with respect to another. The derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ represents the instantaneous rate of change of function f with respect to variable x.

Mathematical Definition:

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Physical Interpretation: In motion analysis, differentiation reveals:

- **Velocity** = rate of change of position: $v = \frac{\mathrm{d}s}{\mathrm{d}t}$
- **Acceleration** = rate of change of velocity: $a = \frac{\mathrm{d}v}{\mathrm{d}t}$
- **Power** = rate of change of energy: $P = \frac{\mathrm{d}E}{\mathrm{d}t}$

Higher-order derivatives provide deeper insights:

$$\text{Position} \xrightarrow{\text{differentiate}} \text{Velocity} \xrightarrow{\text{differentiate}} \text{Acceleration} \xrightarrow{\text{differentiate}} \text{Jerk}$$

Integration: Measuring Accumulation

Integration measures the accumulation of quantities over an interval. The integral $\int f(x)\,dx$ represents the area under the curve $f(x)$ or the total accumulation of f with respect to x.

Mathematical Definition:

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Physical Interpretation: In motion analysis, integration reveals:

- **Displacement** = accumulated velocity over time: $s = \int v\,dt$
- **Velocity** = accumulated acceleration over time: $v = \int a\,dt$
- **Work** = accumulated force over distance: $W = \int F\,ds$
- **Energy** = accumulated power over time: $E = \int P\,dt$

The Fundamental Connection: Integration and differentiation are inverse operations:

$$\text{Acceleration} \xrightarrow{\text{integrate}} \text{Velocity} \xrightarrow{\text{integrate}} \text{Position}$$

A small summary of calculus connections of physical quantites

The kinematic quantities are mathematically related through calculus operations. Differentiation gives the rate of change (velocity from position, acceleration from velocity), while integration reconstructs motion from known rates of change.

$$v = \frac{\mathrm{d}s}{\mathrm{d}t}, \quad a = \frac{\mathrm{d}v}{\mathrm{d}t}, \quad \omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}, \quad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$$

$$v(t) = v_0 + \int_0^t a\,dt, \quad s(t) = s_0 + \int_0^t v\,dt$$

Constant a:

$$v = v_0 + at, \quad s = s_0 + v_0 t + \tfrac{1}{2}at^2, \quad v^2 = v_0^2 + 2a(s - s_0)$$